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Rheology analysis of multiphase flow and its non-viscosity dependent modeling.II: solving and verification

Shuyang Liu[1], Zhihong Han[1]

Abstract. Most multiphase flow model use viscosity as an important parameter, however the viscosity of mixed fluid sometimes is unavailable or difficult to measure especially in small-scale flowing condition. In order to solve this problem, a two-phase wedge-sliding model is developed in this paper and this model is non-viscosity dependent. Two variables of Drift-inhibition angle and expasion-inhibition angle were defined and their expressions were deduced from the model,which can well index the phase-drift trend of mixed fluid. Study also found that the mixed ratio of multiphase fluid has an optimal value, and when the mixed ratio of the heavier phase is less than the optimal one, the flow has better stability.

Key words. Mixed fluid; two-phase wedge-sliding model; phase drift.

1. Introduction

Many multiphase flow models have been constructed by researchers to describe the rheological behavior of mixed fluid. Among them, there are five typical models, such as the Power law model [1], Bingham model[2], Herschel-Bulkley model[3], Casson model[4] and the Ree-Eying model[5] etc., are widely used in industrial field in their respective adaptable conditions. Most of them focus on the dynamic rheological behavior of mixed fluid in a short time, but just little discussion on the flowing stability after a long time. In addition, most of these models regard viscosity as an important even key parameter, but the viscosity of mixed fluid is varying with the temperature, stress and time, and is difficult to measure even unavailable usually.

The more accurate model involves more complex coupling factors, a fast and reasonable computing method is also very important to the practical applicability of the model. In 1692, Higgins and Leighton presented a fast method to calculate thoroughly the performance of two-phase flow in reservoir rock with complex geometry[6], and this method can forecast three-phase flow in complex geometry

[1] Workshop - Jingdezhen Ceramic Institute,Jingdezhen 333403, China

and explain the details by the use of a specific example of a five-spot water flood of a partially depleted stratified reservoir[7]. By using Stokesian Dynamics, Prabhu R. Nott and John F. Brady conducted dynamic simulations of the pressure-driven flow in a channel of a non-Brownian suspension at zero Reynolds number, and in their model, the macroscopic mass, momentum and energy balances are constructed and solved simultaneously[8].

The two-phase wedge-sliding model is characteristic for it's not viscosity dependence, and well prior representation to the stability of mixed fluid, and its practicability still strong relies on a feasible method to solving the complex formula of the model and this method could be easier used in the computer tool.

2. Solving of the wedge-sliding model

As description in the part I of the paper, the moving trend of the heavier phase can be indicated with θ the drift-inhibition angle, and θ_c the expansion-inhibition angle, and both expressions are as following,

$$\theta = \arcsin[\frac{\lambda_f \cdot \lambda_k \cdot C_\rho}{\sqrt{\delta^2 + C_h^2}} \cdot \frac{e^{\lambda_\delta \cdot \delta_y} - e^{-\lambda_\delta \cdot \delta_y}}{e^{\lambda_\delta \cdot \delta_y} + e^{-\lambda_\delta \cdot \delta_y}}] + \theta_c, \theta \in [-\frac{\pi}{2}, +\frac{\pi}{2}] \tag{1}$$

$$\theta_c = \arctan(\frac{C_h}{\delta}) \tag{2}$$

All the definition of the relevant parameters have been given in the part I.

2.1. When a peak stress imposed

Assuming the mixed fluid steadily flows caused by the uniform pressure difference $\Delta P = P_1 - P_2$ between two ends in a slot ,and there is a peak stress $\Delta\delta$ imposed at $x_0(\Delta\delta << \delta)$ shown in Fig.1(b), and $\lambda_i = \lambda_0$,$\theta_i = \theta_0$ before the peak stress added, following equation could be obtained by substituting them into the Eq.(1) in the part I of the paper:

$$f(\delta_i, \theta_i) = (\delta + \Delta\delta) \cdot \sin\theta_0 - \lambda_f \cdot \lambda_k \cdot C_\rho \cdot \frac{e^{\lambda_\delta \cdot \delta_y} - e^{-\lambda_\delta \cdot \delta_y}}{e^{\lambda_\delta \cdot \delta_y} + e^{-\lambda_\delta \cdot \delta_y}} = \Delta\delta \cdot \sin\theta_0 > 0 \tag{3}$$

Because $f(\delta_i, \theta_i) > 0$, so this extra lateral stress will push the B phase of this specific cell to drift along the wedge surface.

If no consideration of the impact from adjacent cells, the drift will cause the volume ratios of B phase in the cell decreases , and eventually cause the decrease of θ_i according to the Eqs.(1) and (3). However, assuming the mixed fluid suits to the optimal design ut supra that $\lambda \in (0, \lambda^*)$, then the increase of θ_i will cause $f(\delta_i, \theta_i)$ decreases and meanwhile lower the drift movement.

Assuming at $\lambda_i^{'}$, the cell achieve its new balance of stress that the extra lateral stress $f(\delta_i, \theta_i^{'}) = 0$ through a time of phase drift, the new drift-inhibition angle can be calculated by the Eq.(**??**)1):

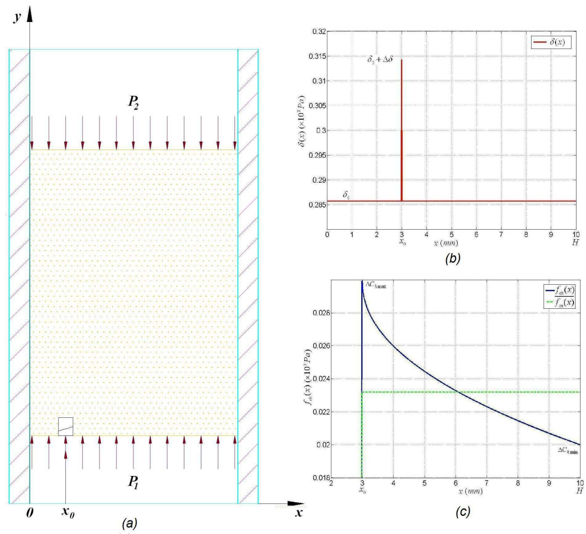

Fig. 1. The stress distribution within mixed fluid caused by a peak stress

$\theta_i^{'} = \arcsin[\frac{\lambda_f \cdot \lambda_k \cdot C_\rho}{\delta + \Delta\delta} \cdot \frac{e^{\lambda_\delta \cdot \delta y} - e^{-\lambda_\delta \cdot \delta y}}{e^{\lambda_\delta \cdot \delta y} + e^{-\lambda_\delta \cdot \delta y}}]$, where $\lambda_i^{'} < \lambda_0$ and $\theta_i^{*} > \theta_0$.

The phases drift of cell S_i will also cause the increase of λ_{i+1} and the balance of stress broken in the adjacent cell S_{i+1}. Thus it is known form the Eq.(3) that the B phase of S_{i+1} will also drift in the same way by the action of the extra lateral stress $f(\delta, \theta_{i+1}) > 0$.

In a horizontal unlimited space, the phases drift will take place in cells successively, and pushes the fluid expanding in the horizontal way, but in slot, the drift movement is restricted and will stop at the wall meanwhile producing a horizontal extra pressure towards the wall. In addition, the drift movement is also restricted by the Brown Movement of the fluid molecules and which will help different phases distributing continuously in the horizontal direction.

So the phase drift which occurs in a single cell will produce a chain of change of the components distribution in the mixed fluid, and an extra horizontal pressure towards the side wall. Defining the extra lateral expansion-inhibition force cause by one cell at x as $C_h = \delta_{21} - \delta_{22}$ and the extra lateral expansion-inhibition force as $\Delta\delta_2$, then $\Delta\delta_2$ can be calculated by:

$$\Delta\delta_2 = \int_{x_0}^{H} C_h(x) \cdot dx \tag{4}$$

It's clearly that $C_h(x)$ is a continuous function in $[x_{0,}H]$, where is the width of the slot.

If taking the horizontal size of cell as $(H - x_0)$, id est. the cell extracted is the transversal section of mixed fluid, no considering the effect of the flow resistance and the plastic deformation of the cell, then the extra lateral expansion-inhibition force cause by peak stress can be obtained theoretically by substituting the Eq.(4) in (1):

$$\Delta\delta_2 \approx \frac{1}{\cos\theta_0} \cdot \left[(\delta + \Delta\delta) \cdot \sin\theta_0 - \lambda_f \cdot \lambda_k \cdot C_{\rho 0} \cdot \frac{e^{\lambda_\delta \cdot \delta_y} - e^{-\lambda_\delta \cdot \delta_y}}{e^{\lambda_\delta \cdot \delta_y} + e^{-\lambda_\delta \cdot \delta_y}}\right] \tag{5}$$

Defining function: $\Delta C_h = f_{ch}(x)$,$x \in [x_0, H]$, its boundary conditions can be gotten from the analysis above: $\Delta C_{h\,\max} = f_{ch}\,|_{x=x_0}$,$\Delta C_{h\,\min} = f_{ch}\,|_{x=H}$, And,

$$R = 1 \tag{6}$$

So,

$$X = Y + 1 \tag{7}$$

Substituting the Eqs. (6) and (7) in the Eq.(4), following function is obtained:

$$\Delta C_{h\,\max} = \frac{H \cdot (\lambda_\delta + 1) \cdot \Delta\delta_2}{(H - x_0) \cdot (H + x_0 \cdot \lambda_\delta)} \tag{8}$$

Then substituting Eqs.(8),(6) and (5) in (7), ΔC_h can be calculated as:

$$\Delta C_h \approx \frac{H\cdot(\lambda_\delta+1)}{(H-x_0)\cdot(H+x_0\cdot\lambda_\delta)} \cdot [1 - \frac{\lambda_\delta\cdot(H-x_0)^{1-\lambda_\delta}\cdot(x-x_0)^{\lambda_\delta}}{H}] \cdot \frac{1}{\cos\theta_0} \cdot [(\delta + \Delta\delta) \cdot \sin\theta_0 - \lambda_f \cdot \lambda_k \cdot C_{\rho 0} \cdot \frac{e^{\lambda_\delta\cdot\delta_y} - e^{-\lambda_\delta\cdot\delta_y}}{e^{\lambda_\delta\cdot\delta_y} + e^{-\lambda_\delta\cdot\delta_y}}] \tag{9}$$

One thing should be pointed that, the Eq.(9) can only qualitatively describe the components distribution in the mixed fluid by action of a peak stress, Id est. the $\Delta\delta_2$calculated by the Eq.(5) is a theoretical result, to get the real value of $\Delta\delta_2$ needs some specific measurement if possible.

The curve of ΔC_h by the action of a peak stress is shown in Fig.6(c), where, $\delta_0 = 0.286 \times 10^3(Pa)$,$H = 10(mm)$,$x_0 = 3(mm)$and $\Delta\delta = \delta/10 = 0.0286 \times 10^3(Pa)$. $\overline{C_h(x)} = \Delta\delta_2/(H - x_0)$ is the mean lateral expansion-inhibition force in the width direction of the slot.

In fact, components drift produces also a lengthways stress to the adjacent cells, and which will cause the gradient variation of the flow velocity in the width direction of the slot. When taking $\delta_0 = 0$, $x_0 = 0$ and $\Delta\delta = \tau$, then the model can be regarded as a flat-flow model of non-Newtonian fluid, and as a special case, all the formula and conclusions above are also suitable too. The higher the lateral loading pressure is, the components drift is more obvious and the stability of the mixed fluid is worse.

2.2. When a gradient face stress imposed

If the force loading on the mixed fluid is a gradient face stress: $\frac{d\delta}{dx} = f_\delta(x)$,$x \in (x_0, H)$

Then the equivalent peak stress at x is:

$$R = X + Y \tag{10}$$

Substituting the Eq.(10) in (9), then the ΔC_h caused by the peak stress can be calculated as:

$$C_h(x) \approx \int_{x_0}^{x} f_{ch}(t) \cdot dt = \frac{H\cdot(\lambda_\delta+1)}{(H-x_0)\cdot(H+x_0\cdot\lambda_\delta)} \cdot [1 - \frac{\lambda_\delta\cdot(H-x_0)^{1-\lambda_\delta}\cdot(x-x_0)^{\lambda_\delta}}{H}] \cdot \frac{1}{\cos\theta_0} \cdot [\delta(x)\cdot\sin\theta_0 - \lambda_f\cdot\lambda_k\cdot C_{\rho 0}\cdot\frac{e^{\lambda_\delta\cdot\delta_y}-e^{-\lambda_\delta\cdot\delta_y}}{e^{\lambda_\delta\cdot\delta_y}+e^{-\lambda_\delta\cdot\delta_y}}] \tag{11}$$

Substituting the Eqs.(10) and (11) in the Eq.(3), following equation is obtained:

$$\delta(x)\cdot\sin\theta_0 - C_h(x)\cdot\cos\theta_0 \approx \lambda_f\cdot\lambda_k\cdot C_{\rho 0}\cdot\frac{e^{\lambda_\delta\cdot\delta_y}-e^{-\lambda_\delta\cdot\delta_y}}{e^{\lambda_\delta\cdot\delta_y}+e^{-\lambda_\delta\cdot\delta_y}} \tag{12}$$

Utilizing the computer tools, the curve of $\theta(x)$ can be discretely drawn by the Eq.(13), and then the drift trend of components of mixed fluid can be reflected by the variation of $\theta(x)$ in the direction of x.

Meanwhile the mean lengthways force and the mean lateral expansion-inhibition force can be calculated respectively by:

$$\overline{\delta(H)} = \int_{x_0}^{H} \delta(x)\cdot dx/(H-x_0), \overline{C_h(H)} = \int_{x_0}^{H} C_h(x)\cdot dx/(H-x_0)$$

Expanding C_ρ of Eq.(12), following equation is obtained:

$$\frac{\rho_B^2+\rho_A^2}{(\rho_B-\rho_A)^2}\cdot[\rho_A+(\rho_B-\rho_A)\cdot\lambda] - \frac{2\cdot\rho_A\cdot\rho_B\cdot(\rho_B+\rho_A)}{(\rho_B-\rho_A)^2} + \frac{2\cdot\rho_B^2\cdot\rho_A^2}{(\rho_B-\rho_A)^2}\cdot\frac{1}{\rho_A+(\rho_B-\rho_A)\cdot\lambda} = \frac{\delta(x)\cdot\sin\theta(x)-C_h(x)\cdot\cos\theta(x)}{\lambda_f\cdot\lambda_k}\cdot\frac{e^{\lambda_\delta\cdot\delta_y}-e^{-\lambda_\delta\cdot\delta_y}}{e^{\lambda_\delta\cdot\delta_y}+e^{-\lambda_\delta\cdot\delta_y}}$$

After deformation:

$$C_1\cdot[\lambda+C_2] - C_3 + C_4\cdot\frac{1}{\lambda+C_2} = \frac{\delta(x)\cdot\sin\theta(x)-C_h(x)\cdot\cos\theta(x)}{\lambda_f\cdot\lambda_k}\cdot\frac{e^{\lambda_\delta\cdot\delta_y}-e^{-\lambda_\delta\cdot\delta_y}}{e^{\lambda_\delta\cdot\delta_y}+e^{-\lambda_\delta\cdot\delta_y}} \tag{13}$$

3. Validation of the model

High temperature grease is a kind of solid or half-solid composition, in which the heavier phases (such as the saponifier, densifier and additives etc.) disperse into the lighter phase(the base oil), and is a typical multi-phases mixed fluid at the high temperature. Cohen and Metzner's study shows that, the flow of mixture in the non-uniform stress field will cause big molecules in it drift towards lower stress side and clear decrease of viscosity near the high stress area[9].Utilizing the two-phase flow model built above, we studied and analyzed the rheological behaviors of high temperature grease to test the validity of the model.

3.1. The distribution of shear stress in pipe

In Fig.2 the front shape description of grease volume in pipe is shown. Studies before [10] found grease flows with a velocity which has a gradient variation at the radius direction of pipe, and the front surface of the grease column bulges. Through the precise fitting and analysis of the curve $\widehat{ABCD}$ shown in Fig.2(b), the shear stress function at the radius direction was obtained as:

$$\tau(r) = \frac{\tau_w}{R - r^*}(r - r^*) \tag{14}$$

Here $r^*$is the radius of the plug flow, and $r^* = 2\tau_y l/\Delta p$.

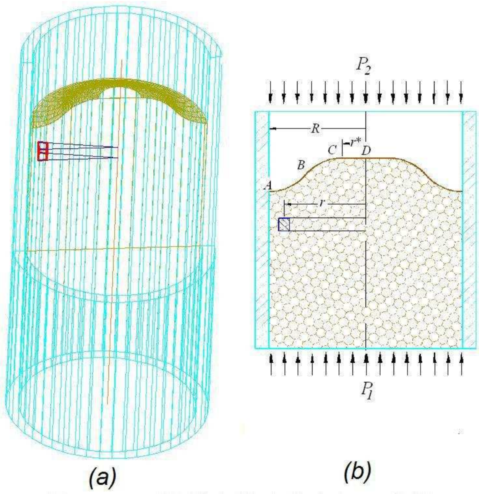

Fig. 2. The front shape of grease volume in pipe

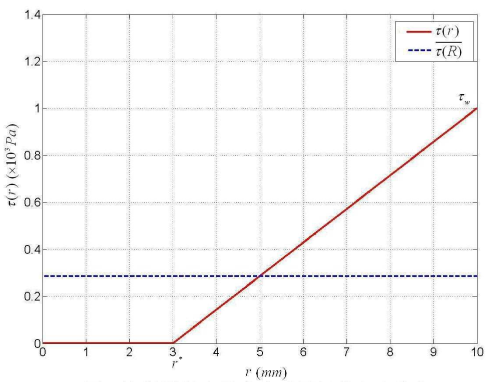

Fig. 3. The distribution of $\tau(r)$ at the radius direction of pipe

Eliminating r^* from Eq.16), then the shear stress function within grease at the radius direction was obtained as follows, and its varying curve is shown in Fig.3.

$$\tau(r) = \begin{cases} \frac{\tau_w}{\Delta P \cdot R - 2\tau_y l} \cdot (\Delta P \cdot r - 2\tau_y l) & (r^* \le r \le R) \\ 0 & (0 \le r \le r^*) \end{cases} \tag{15}$$

Where, L is the radius and length of pipe, l is the length of the grease volume in the pipe, and l is equal to the length of the pipe L when grease is full of the pipe. τ_y is the yield stress of grease, τ_w is the shear stress near the wall of pipe. All of the parameters can be measured through relevant experiments.

3.2. The application of the model

Taking the base oil as A phase, the other components as B phase, and the micro-sector block as cell unit, then ρ_A, ρ_B and Φ_A, Φ_B can be calculated respectively according to the relevant parameters of grease components. Substituting these relevant parameters in the Eqs.(15), C_1, C_2, C_3, C_4, $C_{\rho 0}$ and λ_δ can be obtained respectively.

The δ_y can be measured by single-way compressing experiment, so the λ_k will be gotten.

Taking pipe radius as X-axis and axes as Y-axis, a cylindrical coordinate system is established, shown in Fig.2(b). Measuring the value of the normal pressure of grease upon pipe wall δ_g and noticing that $H = R$ and $x_0 = 0$ because the grease is axisymmetrical in the pipe, the lateral expansion-inhibition force can be calculated by the Eq.(9) as:

$$C_h\left(r\right) = \frac{\left(\lambda_\delta + 1\right)}{R} \cdot \left[1 - \frac{\lambda_\delta \cdot R^{1-\lambda_\delta} \cdot r^{\lambda_\delta}}{R}\right] \cdot \delta_g \tag{16}$$

As for thin tube, it's the difficult to get the value of δ_g by measurement, then it can be obtained by following equation:

$$C_h\left(r\right) \approx \frac{\left(\lambda_\delta + 1\right)}{R} \cdot \left[1 - \frac{\lambda_\delta \cdot R^{1-\lambda_\delta}}{R}\right] \cdot \frac{1}{\cos\theta_0} \cdot \left[\tau\left(r\right)\sin\theta_0 - \lambda_f \cdot \lambda_k \cdot C_{\rho 0} \cdot \frac{e^{\lambda_\delta \cdot \tau_y} - e^{-\lambda_\delta \cdot \tau_y}}{e^{\lambda_\delta \cdot \tau_y} + e^{-\lambda_\delta \cdot \tau_y}}\right] \tag{17}$$

The mean shear stress and the mean lateral expansion-inhibition force respectively are:

$$\overline{\tau\left(R\right)} = \int_0^R \tau\left(r\right) \cdot dr/R = \frac{\tau_w \cdot \left(\Delta P \cdot R - 4 \cdot \tau_y \cdot l\right)}{2\left(\Delta P \cdot R - 2 \cdot \tau_y \cdot l\right)} \tag{18}$$

$$\begin{aligned}\overline{C_h\left(R\right)} &= \textstyle\int_0^R C_h(r) \cdot dr/R \\ &\approx \tfrac{\lambda_\delta + 1}{R} \cdot \left[1 - \tfrac{\lambda_\delta \cdot R^{1-\lambda_\delta} \cdot R^{\lambda_\delta}}{R}\right] \cdot \tfrac{1}{\cos\theta_0} \cdot \left[\overline{\tau\left(R\right)}\sin\theta_0 - \lambda_f \cdot \lambda_k \cdot C_{\rho 0} \cdot \tfrac{e^{\lambda_\delta \cdot \tau_y} - e^{-\lambda_\delta \cdot \tau_y}}{e^{\lambda_\delta \cdot \tau_y} + e^{-\lambda_\delta \cdot \tau_y}}\right]\end{aligned} \tag{19}$$

The internal maximum friction coefficient of grease can be approximately taken for the maximum static friction coefficient between two phases λ_f. No consideration of the gradient variation of stress, the loading forces on the grease are $\overline{\tau(R)}$and τ_y. Because the resistance force of grease are always parallel with the axial of pipe but opposite with the direction of flow, so the micro-stress loading on the cell extracted anywhere should meet the following equation:

$$\left(F_{kA} + F_{kB}\right) + F_f = \overline{\tau(R)} - \tau_y \tag{20}$$

Substituting $C_{\rho 0}$ in the Eq.(22), the following equation is obtained:

$$\lambda_k \cdot (\lambda_f + 1) \cdot C_{\rho 0} \cdot \frac{e^{\lambda_\delta \cdot \tau_y} - e^{-\lambda_\delta \cdot \tau_y}}{e^{\lambda_\delta \cdot \tau_y} + e^{-\lambda_\delta \cdot \tau_y}} = \overline{\tau(R)} - \tau_y$$

After deformation:

3.3. The simulation analysis

Substituting the parameters above in the Eq.(18) or (19), then the discrete values of the lateral expansion-inhibition force $C_h(r)$ can be calculated, and its curve shown in Fig.(9)can also be drawn by use of computer tools. It is known form the Fig.4 that the varying tendency of $C_h(r)$ is same with that of shear stress, and $C_h(R)$ near the wall of pipe obtains the maximum value when r is less than $3mm$.

Substituting the relevant parameters in the Eq.(11), then the discrete values of the drift-inhibition angle $\theta(r)$ can be calculated and its curve shown in Fig.5 can be drawn by use of computer tools, It's clearly shown in the figure that $\theta(r)$ is a decreasing function about r, and its value just indicates the degree of components drift in different area within mixed fluid. Id est. the heavier components are in the biggest tendency to separate from the base oil near the wall, and the shear-thinning phenomenon is more apparent along the direction of stress gradient.

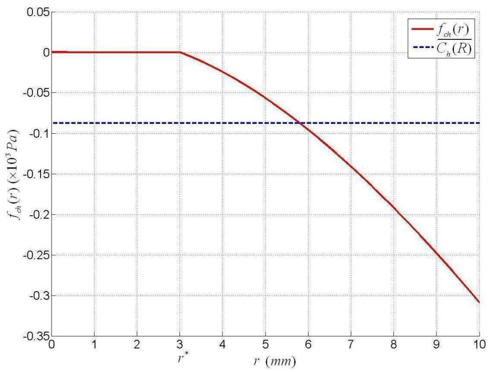

Fig. 4. The distribution of $C_h(r)$ before components drift

Substituting the discrete values of $\theta(r)$ in the Eq.(14) and solving the equations respective, then the discrete value of $\lambda(r)$ can be calculated and its curve can be drawn by the use of computer tools, shown in Fig.(6). It is known form the figure that grease performs a rheological characteristic of Newtonian fluid near the wall of pipe because the fluid there has almost been a single-phase composition.

One thing needs to be pointed out that, although in theory it is regarded there isn't the relative motion within the grease of plug flow area, but the curves above should extend to the full surface of pipe by the role of Brown Movement of the fluid molecules in deed because the variation of $\lambda(r)$ will also cause some change to the front shape of grease volume in pipe.

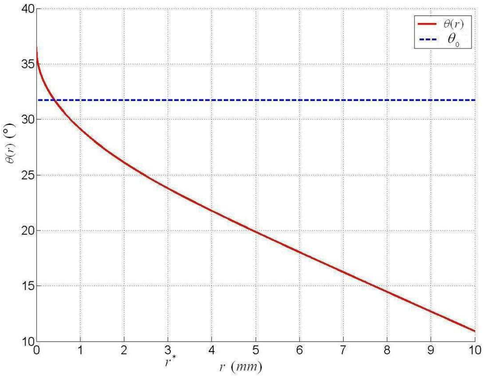

Fig. 5. The distribution of $\theta(r)$ before components drift

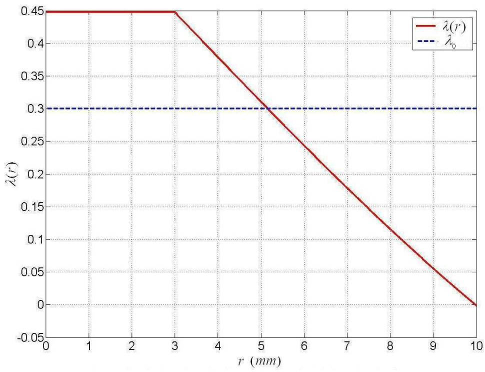

Fig. 6. The distribution of $\lambda(r)$ after components drift

4. Conclusions

(1)Based on the minimum entropy theory of steady-state system, we studied the components drift phenomenon of multiphase mixed fluid in non-uniform stress field, and then proposed a two-phase wedge-sliding model to describe the rheological characteristics of the mixed fluid.

(2) The drift-inhibition angle function $\theta(x)$ was put forward to characterize the drift trend of components along the varying gradient of stress $\delta(x)$ in the paper, and it's proved to be a decreasing function about $\delta(x)$.

(3) The studies show that there is an optimal volume ratio of components λ^* existed where the mixed fluid is most stable, and its theoretical formula is also deduced.

(4) The variation of the lateral expansion-inhibition force $C_h(x)$ along the varying gradient of stress $\delta(x)$ was studied, and its distribution function is given. The analysis following shows that, bigger the value of $C_h(x)$ is, the components of the cell are more tending to separate from each other, and the stability of the mixed fluid is worse.

(5) Applying the model, component-drift phenomena of generalized two-phase mixed fluid on which the loading forces are peak stress and gradient stress were studied respectively, and their rheological equations, drift-inhibition angle equations, and the components distribution functions in the micro-cell $\lambda(x)$ when flow reaches steady state are deduced meanwhile.

(6) Taking the condition of high temperature grease flowing in a pipe as an example and applying this model, the theological behaviors of grease was studied, and the relevant solving procedures were also given in details. The curves of drift-inhibition angle $\theta(r)$ and the components distribution when flow reaches steady state $\lambda(r)$ were drawn by use of computer tools, and the simulation results show that the model can better reveal the rheological behaviors of high temperature grease under gradient stress.

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